Final Exam-Math 201

- Write your name and your I.D. on the booklet
- The duration of the test is two hours
- Calculators are allowed

1. (5 points) Determine whether $\sum_{n=1}^{\infty} \frac{(tan^{-1}n)^2}{n^2+1}$ converge or diverge. Please provide all the details.

2. (10 points) a) Show that for $\mid x \mid < 1$, we have
$$ln\frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + ...\right).$$
(Hint: Determine the expansion of $ln(1+x)$ and $ln(1-x)$ first to determine the expansion of $ln\frac{1+x}{1-x}$).
b) How many terms of the Mclaurin series for $ln(1+x)$ should you add to be sure of calculating $ln1.1$ with an error of magnitude less than 10^{-8}.

3. (10 points) Find the maximum and the minimum values of the function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$ by using the method of Lagrange multipliers.

4. (10 points) Your company manufactures right circular cylindrical molasses storage tanks that are 25 feet high with a radius of 5 feet. How sensitive are the tanks' volumes to small variations in height and radius. After finding the formula for the change in volume, calculate the change in volume if the radius changes from 5 to 5.1 and the height changes from 25 to 24.3.(Hint: The volume of the tank is given by $V = \pi r^2 h$).

5. (5 points) Find all the local extreme values of the function $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$.

6. (10 points)Suppose that $f(x,y,z)$is given by

$$f(x,y,z) = x^3 - xy^2 - z$$

a) Find the rate of change of f at $P(1,1,0)$ in the direction of the vector $v = 2i - 3j + 6k$.

b) In what direction does f change most rapidly at P.

c) Use the linear approximation to obtain an approximate value of f(1.01,1.01,0.1).

7. (10 points)Sketch the regions of integration and evaluate the integrals

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dydx$$

and

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2dydx}{(1+x^2+y^2)^2}$$

8. (5 points) Find the volume of the region in the first octant bounded by the coordinate planes and the planes $x + z = 1$ and $y + 2z = 2$.

9. (5 points) Evaluate $\int\int\int_E \sqrt{x^2+z^2} dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.

10. (5 points) Set up but **do not evaluate** the integral for the volume of the solid that is bounded below by the xy-plane, on the sides by the sphere $x^2 + y^2 + z^2 = 2$ and above by the cone $z = \sqrt{x^2 + y^2}$.

11. (10 points) Given the vector field $F = (e^x cosy + yz)i + (xz - e^x siny)j + (xy + z)k$.

a) Show that F is conservative and find a potential function.

b) Find the work done by the field F along the parabolic path joining the point $(-1,3,9)$ to the point $(1,6,-4)$.

12. (15 points) Given a vector field $F = yi - xj$ and the closed curve C given by parts of the parabola $y = x^2$ and the line $y = 2$. C has counterclockwise orientation and please use the outward normal.

a) Directly evaluate the circulation integral $\int_C F.Tds$,by parametrization.

b) Use Green's theorem to reevaluate the circulation.

c) Use Green's theorem to calculate the flux integral $\int_C F.nds$.